ARTERIAL PRESSURE LOSS FROM VASCULAR VECTOR FLOW MAPPING WITH CONVENTIONAL COLOR DOPPLER

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ABSTRACT

In the presence of a significant arterial stenosis, i.e. a constriction of the arterial lumen, a loss of pressure appears due to the turbulent mixing in the divergent region of the flow. We propose a non-invasive approach, by vascular color Doppler ultrasound, for the measurement of stenotic pressure loss. To this end, we retrieve 1) the transtenotic velocity vector field from conventional color Doppler data, using a constrained optimization method. We then estimate 2) the relative pressure field by combining the Bernoulli and Navier-Stokes equations. To validate this strategy, we developed an in silico pipeline including both fluid dynamics (CFD) and ultrasound imaging (SIMUS software) simulations. As a preliminary test, an in vitro acquisition (color Doppler vs. pressure wire) demonstrated the experimental feasibility of our new method. Our proof-of-concept results tend to demonstrate that it is possible to estimate the pressure losses in arterial stenoses by color Doppler ultrasound.

Index Terms— Arterial stenosis, Pressure loss, Color Doppler, Vector flow imaging, Constrained optimization

1. INTRODUCTION

Assessing the severity of a vascular stenosis is critical for clinical decisions regarding its treatment. The conventional method consists in scanning the impaired region by vascular ultrasound, and in using the NASCET (North American Symptomatic Carotid Endarterectomy Trial) or ECST (European Carotid Surgery Trial) criteria to grade its severity [1]. As known from fluid dynamics basics, these ultrasound indices, based on geometric measurements related to the stenosis, provide little information on its hemodynamic function. A solution is to measure the total pressure loss (or pressure drop) between the inlet and outlet of the stenosis. Such pressure loss is mainly caused by turbulent and wall friction losses that impede flow in the presence of an arterial plaque [2]. Intravascular pressure can be assessed by catheterization, an invasive medical procedure with risk of complications, which consists in inserting a pressure wire into the bloodstream [3]. To limit costs and clinical side effects, noninvasive methods from blood flow imaging would be preferred.

Phase contrast magnetic resonance imaging (PC-MRI) can determine 4-D (3-D + time) blood velocities non-invasively. Starting from PC-MRI-derived velocity vector fields, several investigators have addressed the estimation of pressure drop in arterial [4] or aortic [5] stenoses. The underlying idea is to apply the equations of fluid dynamics to estimate relative pressures. Depending on the hemodynamic conditions, the Navier-Stokes equation or its variants, such as the Bernoulli [6] or Poisson [7] equations, can be used. The time and cost of a PC-MRI scan make it an imaging modality that is primarily intended for clinical research, not routine examinations. In contrast, Doppler ultrasonography is an extensively used clinical modality for real-time blood flow analyses. Whether spectral or color, the conventional Doppler mode provides a scalar field that maps the components of blood velocities along the ultrasound axes. When properly aligned with blood flow, Doppler ultrasound combined with fluid dynamics equation can estimate pressure drops in aortic stenoses [8], or pressure gradients in the left ventricle [9]. To mitigate the beam-to-flow angle issue in color Doppler, several techniques have been proposed to derive vector velocity fields, especially in ultrafast ultrasound [10, 11]. In particular, Olesen *et al*. [12] used transverse oscillations for vector flow imaging, then estimated pressure differences in *in vitro* arterial stenoses through the Bernoulli principle. The Bernoulli equation, however, does not consider pressure losses as there is no viscosity term. Only the static and dynamic pressure conversions can be measured with non-viscous fluids; the irreversible pressure losses due to viscous forces and turbulent mixing cannot be determined.

In this study, we aimed at estimating turbulence-induced pressure drops in arterial stenoses from conventional clinical color Doppler. Our contribution was twofold. 1) Inspired by 2D-*i*VFM (intraventricular vector flow mapping) [13, 14], we developed a 2D-*v*VFM (vascular vector flow mapping) technique based on color flow imaging. This approach allowed us to estimate intravascular velocity vector fields from color Doppler images. 2) We then estimated the pressure drops resulting from the stenoses by combining the Bernoulli and Navier-Stokes equations. The innovative method was validated through fluid dynamics (CFD) and ultrasound (SIMUS) simulations in axisymmetric models of carotid stenosis. We

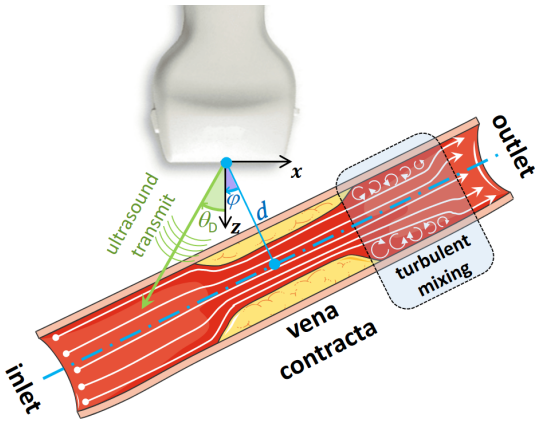

Fig. 1. Color Doppler ultrasound in a quasi-axisymmetric arterial stenosis. θ_D is the Doppler pulse-echo angle. d is the distance between the transducer center and the vessel axis.

then tested its feasibility in an *in vitro* phantom. In the following, we describe the theoretical background, as well as the *in silico* and *in vitro* methods and results.

2. METHOD AND MATERIALS

2.1. The 2-D *v*VFM optimization problem

The objective is to recover the 2-D velocity vector field $\vec{v} = \{v_x, v_z\}$ from the scalar Doppler field u_D. The 2D-*v*VFM problem for axisymmetric stenoses was written in a right-handed Cartesian grid associated to a linear transducer. We note: θ_D, the Doppler pulse-echo angle, and φ, the angle of the vessel axis to the transducer, both about the y axis; d, the distance between the vessel axis and the center of the transducer (Fig. 1). By convention, u_D is positive when flow goes towards the transducer, and the angles are defined counter-clockwise (trigonometric direction). Doppler velocities are written as:

$$u_D = -\sin(\theta_D)\, v_x - \cos(\theta_D)\, v_z. \tag{1}$$

The vector velocity field $\vec{v} = \{v_x, v_z\}$ is estimated by minimizing the cost function

$$J(\vec{v}) = \int_\Omega \left(u_D + \sin(\theta_D)\, v_x + \cos(\theta_D)\, v_z\right)^2 + \alpha \mathcal{L}(\vec{v})\ \mathrm{d}\Omega \tag{2}$$

subject to the following constraints:

$$\begin{aligned}(-x\sin\varphi + z\cos\varphi - d)\,\mathrm{div}\vec{v} = \\ (-x\sin\varphi + z\cos\varphi - d)\,\partial_z v_z - v_x\sin\varphi + v_z\cos\varphi \\ + (-x\sin\varphi + z\cos\varphi - d)\,\partial_x v_x = 0 \ \text{ on } \Omega,\end{aligned} \tag{3}$$

$$\vec{v}.\vec{n}|_{wall} = v_x n_x + v_z n_z = 0 \ \text{ on } \partial\Omega. \tag{4}$$

The term $\mathcal{L}(\vec{v})$ in Eq. (2) contains second-order partial derivatives of the velocity with respect to x and z. It is tuned by α, a small positive scalar, and induces spatial smoothing. The first constraint (3) enforces mass conservation (divergence-free flow). The multiplicative term ensures numerical stability; it represents the distance from the main axis. The second constraint (4) imposes tangential boundary conditions. Ω indicates the intravascular region of interest, and $\partial\Omega$ stands for its boundary. The unit vector normal to the vessel walls is noted $\vec{n}|_{wall} = \{n_x, n_z\}$. This constrained least-squares problem was solved by using the method of Lagrange multipliers, as in [14]. A finite-difference discretization of the optimization problem led to a full-rank sparse symmetric linear system that returned the velocity estimates $\{v_{x-v\mathrm{VFM}}, v_{z-v\mathrm{VFM}}\}$.

2.2. Fluid-dynamics and ultrasound simulations

To validate the proposed method, we simulated blood flow and color Doppler in axisymmetric models of carotid stenoses (8-mm inner diameter). Two different stenoses (Cosine- and Gaussian-shaped) were created, each with two degrees of stenosis: 60% and 70% in diameter reduction, which reflected moderate and severe stenoses. For each configuration, three steady flow rates were studied (0.3, 6, 1 L/min) by using COMSOL Multiphysics (COMSOL, Inc). The fluid was Newtonian and the regime was turbulent ($k-\epsilon$ model). The simulated flow fields were seeded with ~50,000 pseudo-randomly distributed point scatterers. We then performed color Doppler simulations using SIMUS (simulator for ultrasound imaging, [15, 16]) with the following properties: 128-element 7.6-MHz linear array, focused transmits with sliding sub-apertures of 16 elements, beam-to-flow angle of 15°, Doppler packet size of 6. The simulated ultrasound signals were post-processed (demodulation, beamforming, auto-correlation) using the MATLAB ultrasound MUST toolbox [17] to obtain Doppler velocities on a Cartesian grid. The velocity vector fields were then estimated by using the above-mentioned *v*VFM method, and compared against the original CFD velocity fields: $\{v_{x-v\mathrm{VFM}}, v_{z-v\mathrm{VFM}}\}$ vs. $\{v_{x-\mathrm{CFD}}, v_{z-\mathrm{CFD}}\}$.

2.3. Relative pressures and pressure drops

In the presence of a constriction, the flow converges down to the *vena contracta*, then diverges to recover its initial geometry. Most of the energy loss occurs in the divergent zone, as a result of turbulent mixing. We estimated the pressure drops ($\Delta P = P_{\mathrm{inlet}} - P_{\mathrm{outlet}}$) by splitting the geometry into two sections. 1) We used the Bernoulli equation (no energy dissipation) between the inlet and the *vena contracta* to account for the static-to-dynamic pressure conversion. 2) We integrated the Navier-Stokes equation between the *vena contracta* and the outlet by using a finite-difference scheme. The estimated pressure drops were compared against the actual

losses: $\Delta P_{v\text{VFM}}$ vs. ΔP_{CFD}. From the inlet ($_{\text{in}}$) to the *vena contracta* ($_{\text{vc}}$), the velocity-pressure relationship was thus written as

$$P_{\text{in}} - P_{\text{vc}} = \frac{1}{2}\rho\left({V_{\text{vc}}}^2 - {V_{\text{in}}}^2\right) + \rho\int_{\text{in}}^{\text{vc}} \frac{\partial V}{\partial t}\,\mathrm{d}l. \qquad (5)$$

Neglecting the effect of gravity, the velocity-pressure relationship, for a Newtonian incompressible fluid, from the *vena contracta* to the outlet read

$$\nabla P = \mu\nabla^2\vec{V} - \rho\left(\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V}\right). \qquad (6)$$

The variables ρ, P, μ, V, and t stand for fluid density, pressure, dynamic viscosity, velocity, and time, respectively. Note that we have only studied steady flows so far. The inertial terms were therefore zero.

2.4. *In vitro* analysis

To test the practical feasibility of pressure drop estimation by the 2D-*v*VFM technique, we performed *in vitro* echographic experiments on an in-house stenotic phantom. The fluid (water) was seeded with Polyamid particles (50 μm in diameter) to ensure ultrasound backscattering. The stenosis was Cosine-shaped with a 60%-diameter occlusion and the steady flow rate was 1 L/min. We measured the pressures at the inlet, *vena contracta*, and outlet with a ComboWire XT guide wire (Philips). We used a GE Vivid iq Premium ultrasound scanner (GE Healthcare) with a GE 9L linear probe to measure the color Doppler velocities. The relative pressures and pressure drops were estimated by *v*VFM, as explained above, and compared against the ground-truth (wire) values.

3. RESULTS

3.1. *In silico* results

Fig. 2 illustrates one example of simulated color Doppler with the corresponding *v*VFM-derived velocity vector field compared with the CFD-based ground-truth field. The normalized root mean square errors (nRMSE, estimation vs. ground-truth) with respect to the flow velocity are listed in Table 1. The nRMSE values in the *x*- and *z*- directions were smaller than 6.1% and 1.5%, respectively. These results tend to show that our *v*VFM technique can accurately estimate velocity vector fields from Doppler velocities in arterial stenoses. Fig. 3 shows how the *v*VFM-derived pressure drops compared with the actual pressure losses calculated by CFD. The log-based regression line was $y = 1.01x - 0.02$, with a coefficient of determination of $r^2 = 0.96$, which tends to confirm the validity of the proposed non-invasive method.

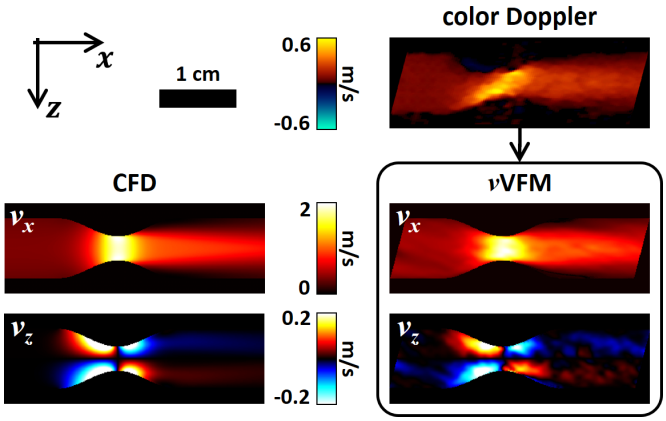

Fig. 2. Realistic simulation of color Doppler imaging in an axisymmetric stenosis (top right). The *v*VFM method returns the vector components v_x, v_z (right column). The original CFD components are given (left column) for a comparison.

shape/degree	velocity component	flow rate [L/min]		
		0.3	0.6	1
Cosine 60%	v_x	6.0	4.5	5.3
	v_z	1.2	1.0	1.2
Cosine 70%	v_x	4.6	4.7	4.8
	v_z	1.0	1.0	1.0
Gaussian 60%	v_x	4.7	6.1	5.0
	v_z	1.5	1.5	1.3
Gaussian 70%	v_x	3.8	4.8	4.1
	v_z	1.1	1.2	1.0

Table 1. *In silico* study — Normalized root mean square errors (nRMSE in %) between the *v*VFM-derived and CFD-based velocities in the *x* and *z* directions. We simulated four carotid stenoses and three flow rates.

3.2. *In vitro* results

Fig. 4 illustrates the result of the *in vitro* experiment on the carotid stenosis phantom. The velocity vector field (top) was obtained by *v*VFM from conventional color Doppler imaging. The red curve (bottom) represents the relative pressures estimated from the *v*VFM field along the vessel axis. As a comparison, the three points give the invasive values measured by the pressure sensor. This result indicates the feasibility of the non-invasive method in experimental flow conditions.

4. DISCUSSION AND CONCLUSION

We focused on the hemodynamic assessment of arterial stenosis by ultrasound imaging. The innovation was twofold: 1) We introduced a vector flow method called vascular vector flow mapping (*v*VFM), inspired by the *i*VFM dedicated to the intraventricular flow [14]. 2) From the recovered velocity vector fields, we introduced a method to estimate the pressure

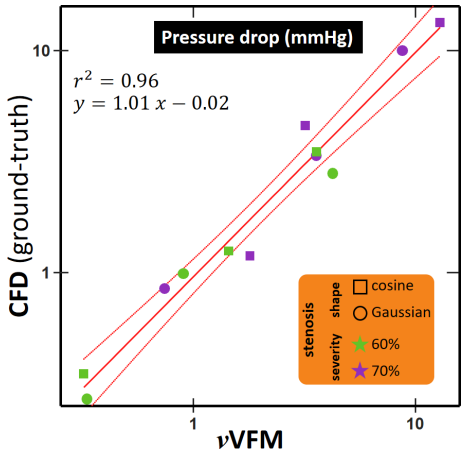

Fig. 3. *In silico* study — Comparison of the *v*VFM-derived pressure drops with the actual CFD pressure drops (in a logarithmic scale to get a uniform distribution). We ran 12 simulations: two stenosis shapes, two degrees of severity, three flow rates.

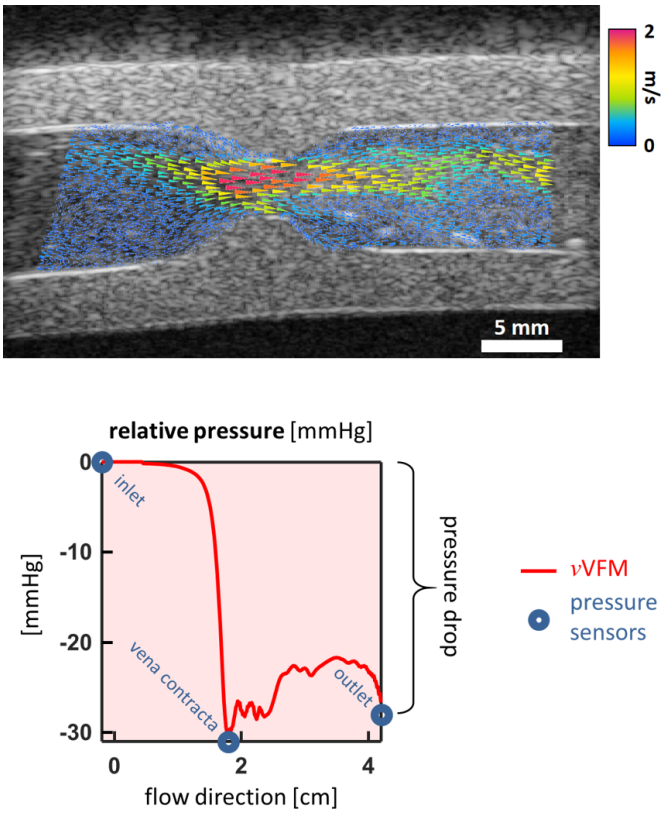

Fig. 4. *In vitro* study — Top: Vector flow map recovered by *v*VFM in a Cosine-shaped 60%-stenosis. Bottom: Relative pressures along the vessel axis derived by *v*VFM compared with the invasive measurements.

loss that occurs in the turbulent mixing of the diverging flow.

Vector flow mapping. The *v*VFM technique that we introduced allows one to recover a 2-D velocity vector field of the intravascular flow from conventional color Doppler imaging. Assuming that the flow is quasi-axisymmetric, *v*VFM retrieves the two components of the velocity in the longitudinal plane by using the mass conservation and tangential boundary conditions. These physical constraints make the least-squares optimization problem well-posed and help to obtain a consistent flow field, as confirmed by the CFD+SIMUS results. It should be noted that velocity vector fields can also be obtained in high-frame-rate ultrasound by transmitting wide wavefronts [10]; for example, by beamforming backscattered echoes at two different angles [11]. However, unlike the *v*VFM technique, these methods cannot currently be implemented in clinical ultrasound scanners.

Estimation of the pressure loss. The Bernoulli equation assumes that the fluid is inviscid. Hence, it cannot compute pressure losses. Only static-dynamic pressure conversions can be evaluated [12, 18] as long as viscous losses are negligible (e.g. upstream from the stenosis). Awareness of the pressure drop associated with the presence of an arterial stenosis is clinically relevant. Accounting for viscous terms downstream of the *vena contracta*, the *v*VFM method, together with the Bernoulli and Navier-Stokes equations, allowed us to estimate pressure drops with good accuracy. Our preliminary results thus demonstrate that noninvasive pressure drop measurements could be possible in a clinical setting.

Limitations and perspectives. In this paper, we have addressed nearly axisymmetric steady flow conditions. The study will be generalized to pulsatile flows by averaging the equations temporally during one or more cardiac cycles. We will also need to evaluate the effect of geometry asymmetry on pressure drop estimates, as well as the effect of misalignment of the ultrasound probe with the flow axis. Note however that the flows were not asymmetric (Fig. 4) due to experimental hazards (stenosis phantom not perfectly straight, wall roughness, flow instability by turbulence, upstream flow not fully developed, etc.). This tends to show that our method can also work in non-axisymmetric flows. These analyses will be performed through simulations and *in vitro* experiments. Once validated, the clinical relevance of manometric measurements by *v*VFM will then be investigated in a clinical context.

Conclusion. Measurement of pressure losses generated by a vascular stenosis can be achieved by Doppler ultrasound. This innovative method could have important clinical implications for treatment decisions in patients.

5. COMPLIANCE WITH ETHICAL STANDARDS

This paper reports no data from studies involving human and/or animal subjects.

6. ACKNOWLEDGMENTS

This work was supported by the LabEx CeLyA (ANR-10-LABX-0060), the LabEx PRIMES (ANR-11-LABX-0063), and EUR MANUTECH SLEIGHT (ANR-17-EURE-0026), within the program "Investissements d'Avenir" (ANR-11-IDEX-0007) operated by the French National Research Agency (ANR). This material is based upon work done on the PILoT facility (INSA Lyon, France).

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